ESTIMATION OF FUNCTIONAL BRAIN CONECTIVITY

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ABSTRACT

This article presents an overview about the basis of estimators of connectivity, such as Directed Transfer Function (DTF) and Partial Directed Coherence (PDC), their differences and their applicability in the estimation and analysis of functional brain connectivity.

I. INTRODUCTION

It is well known that behind every our behaviour, physical or mental, specific brain regions are activated. During execution of a task, different activities are performed simultaneously therefore different brain regions are evoked. Neuroimaging concerns with building brain maps that reveal where the cortical activations appear during the execution of a task. For many years, the monitoring of the electrical signals derived from the brain regions have been used for inferring functional aspects of many normal and pathological brain processes. The central question is does or how these regions that are involved in the task, cooperate one to each other during the execution of the task. How to define the information flow between the cortical activities? Is there any connectivity pattern between the brain regions? Different types of connectivity are defined. Anatomical connectivity represents the existence of anatomical links allowing the information flow from a cerebral district to another one. Effective connectivity represents the simplest brain circuit that would produce the same temporal relationship as observed experimentally between cortical sites. Functional connectivity represents the existence of temporal correlation / coherence between the brain activity recorded in different cerebral sites. In this article we concern with the techniques and estimators that are used to obtain functional connectivity.

Norbert Wiener defined causality for the first time. Given two simultaneously measured signals, if one can predict the first signal better by incorporating the past information from the second signal than using only information from the first one, then the second signal can be called causal to the first one (Wiener, 1956). The economist Clive Granger [1] in 1969 gives mathematical formulation of the Wiener definition and introduces the concept of Granger Causality, which we refer later in this article. Kaminski and Blinowska have introduced the Directed Transfer Function [2, 3] which can be used to determine the directional influences between any given pair of channels in a multivariate dataset. Another estimator, Partial Directed Coherence is introduced by Baccala and Sameshima [4] as a factorization of the partial coherence. This estimator is of particular interest in applications to brain signals. Both of these parameters include the concept of Granger Causality in them and in this article we stress the differences between the two. Also, a time-varying functional cortical connectivity can be obtained using these parameters [5]. In that way, the whole communication between the brain regions can be monitored and described during the execution of a given task. Using these parameters for obtaining functional connectivity, many applications in clinical neurophysiology have been obtained [6, 7, 8] that explain some neurophysiological behaviours.

II. AUTOREGRESSIVE MODELING AND GRANGER CAUSALITY

If we have a time series of a signal, one can try to predict the future values of the signal using an autoregressive filter as linear predictor. The linear predictor is defined as:

$$\bar{x}(n) = -\sum_{k=1}^{p} a(k)x(n-k) \tag{1}$$

The goal is to determine the coefficients *a(k)* by minimizing the power of the prediction error:

$$e(n) = x(n) - \bar{x}(n) \tag{2}$$

Given two time series *a(t)* and *b(t)*, *a(t)* is said to Granger-cause *b(t)* if the insertion of *a(t)*'s past into an autoregressive modelization of *b(t)* significantly improves the prediction of *b(n)*, that is, if it reduces its prediction error. By means of bivariate autoregressive modelling of *a(t)* and *b(t)*: *a(t)* is said to Granger-cause *b(t)* if by inserting *a(t)*'s past samples in the autoregressive modelization of *b(t)* this can reduce the prediction error. Here we have directionality of the form *a(t)→b(t)*, or *a(t)* Granger-causes *b(t)*. It can be *a(t)→b(t)* without necessarily being *b(t)→a(t)*.

A. Bivariate modeling

Let x(t) and y(t) denote the time series from two data channels. Suppose that the temporal dynamics of x(t) and y(t) are given by the following bivariate autoregressive relations:

$$x(n)=\sum_{k=1}^{p}a_{xx}(k)x(n-k)+\sum_{k=1}^{p}a_{xy}(k)y(n-k)+e_{xy}(n)$$
$$y(n)=\sum_{k=1}^{p}a_{yx}(k)x(n-k)+\sum_{k=1}^{p}a_{yy}(k)y(n-k)+e_{yx}(n) \tag{3}$$

Consider the univariate case, when we describe the signals using the autoregressive modelling:

$$x(n)=\sum_{k=1}^{p}a_x(k)x(n-k)+e_x(n)$$
$$y(n)=\sum_{k=1}^{p}a_y(k)x(n-k)+e_y(n) \tag{4}$$

In the univariate case, a_x and a_y are the model parameters, p is the model order and e_x and e_y are the uncertainties or the noises associated with the model. Here, the prediction error depends only on the past values of the own signal. Now consider the bivariate case. Here, the prediction error for each individual signal depends on the past values of both signals. The performances of the prediction for both models can be assessed with the variances of the prediction errors. Let var(e_x) and var(e_y) be the variances of the prediction errors of *x(t)* and *y(t)* respectively for the univariate case, and var(e_{xy}) and var(e_{yx}) the variances of the prediction errors of *x(t)* and *y(t)* respectively for the bivariate case. The measure of Granger causality from *y* to *x* can be expressed as

$$G_{y\to x}=\ln\frac{\mathrm{var}(e_x)}{\mathrm{var}(e_{xy})} \tag{5}$$

If the past value of *y* does not improve the prediction of *x*, then $var(e_x)\approx var(e_{xy})$ and $G_{y\to x}=0$. Any improvement of the prediction of *x* by inclusion of *y* makes $var(e_{xy})\downarrow$, therefore $G_{y\to x}\uparrow$. Similarly, the measure of Granger causality from *x* to *y* can be defined as:

$$G_{x\to y}=\ln\frac{\mathrm{var}(e_y)}{\mathrm{var}(e_{yx})} \tag{6}$$

B. Multivariate modeling

If we have more than two data channels, one can use bivariate methods for estimating the causality links between the signals. Namely, a bivariate model would be obtained for every pair of data channels. But when we have multiple signals, bivariate methods have limitations. Consider the following scenario with three signals: signal 3 causes signal 1 and signal 2. Or in other words, the causality links that exist are 3→1 and 3→2. If we use the bivariate method for modelling the influence signal 1 has on signal 2, than the connectivity pattern obtained would be that there is a casual link between these signals, or there exist 1→2. The model does not recognize that this influence is because of the signal 3. The model is not aware of the common effect of signal 3. Multivariate method builds a unique model so that the connectivity pattern that is obtained takes into account all the signals in the set and all their interactions. Multivariate methods by building a unique model that is based on all the signals use all the information at disposal and allow a better comprehension of the relationship between the signals. Given the set of signals:

$$\overline{X}=[x_1,x_2......x_N] \tag{7}$$

The multivariate autoregressive model of order p is:

$$x_1(n)=\sum_{k=1}^{p}a_{11}(k)x_1(n-k)+...+\sum_{k=1}^{p}a_{1N}(k)x_N(n-k)+e_1(n)$$
$$x_2(n)=\sum_{k=1}^{p}a_{21}(k)x_1(n-k)+...+\sum_{k=1}^{p}a_{2N}(k)x_N(n-k)+e_2(n)$$
$$\vdots$$
$$x_N(n)=\sum_{k=1}^{p}a_{N1}(k)x_1(n-k)+...+\sum_{k=1}^{p}a_{NN}(k)x_N(n-k)+e_N(n) \tag{8}$$

The model parameters are the N variances of the noises:

$$S_N=\begin{bmatrix}\sigma_1\\ :\\ \sigma_N\end{bmatrix} \tag{9}$$

And the coefficients *a(k):*

$$\overline{a}(1)=\begin{bmatrix}a_{11}(1) & ... & a_{1N}(1)\\ : & : & :\\ a_{N1}(1) & ... & a_{NN}(1)\end{bmatrix}\quad \overline{a}(N)=\begin{bmatrix}a_{11}(N) & ... & a_{1N}(N)\\ : & : & :\\ a_{N1}(N) & ... & a_{NN}(N)\end{bmatrix} \tag{10}$$

III. Direct Transfer Function and Partial Directed Coherence

One of the limitations of the causality estimation is the time domain of the signals. Given the signals in the time domain, the time window used to identify the model provided that the signals are stationary in that time window. The frequency domain contains more information about the signal, as how much of the signal lies within each given frequency, so it is practical to obtain pattern of connectivity in the frequency domain.

A. MVAR in frequency domain

To obtain the multivariate autoregressive (MVAR) model in the spectral domain we Fourier transform equation (1) to obtain:

$$\overline{A}(f)\overline{X}(f)=\overline{E}(f) \tag{11}$$

Where:

$$A_{ij}(f) = \sum_{k=0}^{p} a_{ij}(k) e^{-j2\pi fTk}$$

$$\overline{A}(f) = \begin{bmatrix} A_{11}(f) & \dots & A_{1N}(f) \\ : & : & : \\ A_{N1}(f) & \dots & A_{NN}(f) \end{bmatrix}$$

$$\overline{X}(f) = \begin{bmatrix} X_1(f) \\ : \\ X_N(f) \end{bmatrix} . \overline{E}(f) = \begin{bmatrix} E_1(f) \\ : \\ E_N(f) \end{bmatrix}$$

B. Direct Transfer Function - DTF

If we continue further to solve matrix equation (11) to obtain *X(f)*, we get:

$$\overline{X}(f) = \overline{A}^{-1}(f)\overline{E}(f) = \overline{H}(f)\overline{E}(f) \tag{12}$$

Where:

$$\overline{H}(f) = \overline{A}^{-1}(f) = \begin{bmatrix} H_{11}(f) & \dots & H_{1N}(f) \\ : & : & : \\ H_{N1}(f) & \dots & H_{NN}(f) \end{bmatrix}$$

Is the transfer matrix of the MVAR filter. On the basis of matrix *H(f)*, Blinowska and Kaminski [2] defined the Directed Transfer Function (DTF) from *j* to *i* as:

$$\theta_{ij}(f) = \left| H_{ij}(f) \right|^2 \tag{13}$$

Since, $H_{ij} \neq H_{ji}$, it is obvious that $\Theta_{ij} \neq \Theta_{ji}$ also. The value of DTF_{ij} at a certain frequency f_0 represents the existence of a causality link directed from *j* to *i*.

C. Partial Directed Coherence - PDC

Baccala and Sameshima [4] defined Partial Directed Coherence (PDC) on the basis of matrix *A(f)* from equation (11):

$$\pi_{ij}(f) = \left| A_{ij}(f) \right|^2 \tag{14}$$

Similar as with the DTF, since $H_{ij} \neq H_{ji}$, $\Theta_{ij} \neq \Theta_{ji}$ also. The value of PDC_{ij} at a given frequency represents the exsistence of a causality link directed from *j* to *i.*

As it can be seen, the mathematical definition of DTF and PDC is different. Due to their mathematical formulation and matrix inversion, there are some differences between these two parameters. DTF describes the sum of all influences (direct and indirect) directed from *i* to *j*. PDC describes only direct influences. That means if there is no direct influence between two signals, DTF can still have some significant value, while PDC does not. Depending on the kind of information we are interested in, we can decide which one we need to use.

IV. Discussion

After we pointed the two popular estimators of connectivity, it is time to address their appliance in neuroscience. With the help of electroencephalogram (EEG) one can measure the brain signals that correspond to the patient current behaviour. The EEG cap is consisted of certain number of electrodes. Each electrode is placed on top of a certain brain region and records the potential from the activity that is present in that brain region. Each electrode is source of an electric signal, so the number of model parameters depends on the number of electrodes placed on the patient's head. Furthermore, when investigating some specific behaviour, one can choose specific regions of interest, and take into account the signals that are generated from the electrodes on top of these regions. Once we obtain the connectivity pattern between the regions of interest, we can observe the information flow between brain regions during the state in which the patient is recorded. Moreover, once we got the connectivity pattern, one can build a directed graph on it, and use the powerful tools form graph theory to calculate some graph parameters who can describe somehow describe the process that is taking place on the specific network.

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