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Lindenmayer System Application on the Human Kidney Arterial System

Ǵorǵevik D, Loškovska S and Mihajlov D

Electrotechnical Faculty, St. Cyril & Methodius University, Skopje, Macedonia

A Lyndenmayer system application to the human body organ system is presented. Stochastical L-systems extended with stochastical functions, as a mechanism for obtaining model variations, and extended 'turtle' logic for graphics presentation of obtained pictures are described. As an example, an L-system application to the human kidney arterial system is presented together with all model modifications.

1. Introduction

In 1968 Danish biologist Astrid Lindenmayer [1] introduced a concept of Lyndenmayer systems (L-systems) in order to obtain a tool for the automatic generation of pictures of plant structures. Since then, in addition to plant pictures, L-systems have been used for generating pictures of fractal curves, reproduction of a traditional East Indian art forms, graphically motivated algorithms for music generation, as well as for generating pictures of human body organ systems with branching anatomic structures (e.g. lung tracheo-bronhial system, cardiovascular and lymph systems and, the urinary tract).

A program for modeling human body organ systems following an L-system approach is developed. The basic L-system approach is extended with rules necessary to obtain common anatomic characteristics in each system. Therefore stochastic functions in L-system production rules are used for modeling variations which appear in particular samples of organ systems.

The paper describes a stochastic L-system concept and an extended 'turtle' logic for graphic interpretation of the models. To exemplify the program possibilities, an application of L-system on the human kidney arterial system is presented. Furthermore, it discusses the modeling of vascular anatomy variations of a healthy kidney, kidney with infarct and arterio-sclerotic kidney, with a developed mathematical model.

2. Stochastic L-systems

The basic idea of L-systems is to define complex objects by successively replacing parts of a simple initial object using a set of rewriting rules. L-system is defined as follows:

Let a nonempty set V denote an alphabet, and V^* denote the set of all words over V. An L-system is an ordered triplet $G = < V, W, P >$ where:

- V is the alphabet of the system;
- $W \in V^*$ is a nonempty word called axiom; and
- $P \subset V \times V^*$ is a finite set of productions.

A pair (α, χ) is a production, if for any letter $a \in V$, there is at least one word $\chi \in V^*$, such $\alpha \rightarrow \chi$. The letter α and the word χ are called the predecessor and the successor respectively.

Let $M=\alpha_1\alpha_2...\alpha_m$ be an arbitrary word over V^*. The word $N=\chi_1\chi_2...\chi_n \in V^*$ is directly derived (or generated by) from M (then we write $M \Rightarrow N$) if and only if $\alpha_l \rightarrow \chi_l$, for all l=1,...,m. A word N is generated by G in a derivation length n if there exist a sequence of words, $M_0, M_1, M_2, \ldots, M_n$ such that $M_0=W, \ldots, M_n=N$ and $M_0 \Rightarrow M_1 \Rightarrow \ldots \Rightarrow M_n$.

A stochastic L-system is an ordered quadruplet $G_\pi = < V, W, P, \pi >$. The alphabet V, the axiom W and the set of productions P are defined above. Function: π: $P \rightarrow (0,1]$, called probability distribution, maps the set of productions into the set of production probabilities. It is assumed that for any letter $a \in V$ the sum of probabilities of all productions with predecessor a is equal to 1. A general format of a stochastic L-system production rule is:

$$\text{predecessor}(\alpha) \xrightarrow{\pi_1} \text{successor}(1)$$
$$\text{predecessor}(\alpha) \xrightarrow{\pi_2} \text{successor}(2)$$
$$\ldots$$
$$\text{predecessor}(\alpha) \xrightarrow{\pi_n} \text{successor}(n)$$

where $\sum \pi_i = 1$.

3. Extended L-system graphics presentation

An extended turtle geometry for graphical interpretation of strings is used. A state of the turtle is defined as a triplet (x,y,α), where:

- (x,y) are Cartesian coordinates of the turtle's position;
- angle α called the turtle heading is interpreted as the direction the turtle is facing.

For a given step size d and angle increment δ, the turtle responds to the commands represented by the following symbols:

F move forward a step of length d. The state of the turtle changes to (x',y',α) where:

$$x'=x+d\cdot\cos(\alpha)$$
$$y'=y+d\cdot\sin(\alpha).$$

A line segment between points (x,y) and (x',y') is drawn.

f move forward a step of length d without drawing a line.

\+ turn right by angle δ. The next state of the turtle is $(x,y,\alpha+\delta)$.

\- turn left by angle δ. The next state of the turtle is $(x,y,\alpha-\delta)$.

[push the current state of the turtle onto a pushdown stack. The information saved on the stack contains the turtle's position and orientation, as well as other attributes such as the color and width of line being drawn.

[pop a state from the stack and make it the current state of the turtle. No line is drawn, although in general the position of the turtle changes.

!/$ changes the line segment width.

:/; changes the line segment length.

'/` changes the line segment color.

4. L-system model of the human kidney arterial system

The program accepts as input the mathematical model of the organ system. The following model is used for generating the kidney arterial system picture:

```
derivation length: 16
angle factor: 24
```

```
start fatness: 38
fatness increment: *2
length increment: *2
scale factor: 100
axiom: '''''::F;;!![apical][lower]::$L!;;[```posterior][upper][middle]
ignore: +-
(segmental arteries)
posterior --> D  A
apical    --> D  B
lower     --> D  D
upper     --> D  C
middle    --> D  G
A --> D ::LLL;;J
B --> D ++++L++a
C --> D ++::LLLp
D --> D ----L--a
G --> D --::LLLp
(stochastic branching of the segmental arteries on 3 to 5 interlobar arteries)
a --> U JJJKK
J --> D !:[Lb][+++Lb][---Lb][+++++Lb][-----Lb]
K --> D !:[++Lb][-Lb][++++Lb][---Lb]
p --> L qsr
q --> D ![LL;b][+++L-L;b][---L+L;b][+++++L:L;--L;b][--L:L;++L;b]
r --> D ![++LL;b][-LL;b][++++L-L;b][---L+L;b]
s --> D ![LL;b][---LL;b][+++L-L;b]
(branching of the interlobar arteries on 2 arcuatae arteries)
b --> U EN
E --> D ::::![++++++c][------d]
N --> D :::![+L+++++:c][-L-----:d]
c --> D IO
d --> D HP
O --> D IQ
P --> D HR
Q --> D I
R --> D H
(branching of the interlobular and medular arteries)
H --> D L[++++++!LX]L![------LT][++++++LX]$
I --> D L[------!LX]L![++++++LT][------LX]$
X --> D gX
(branching of the arteries which end in the Malpigian bodies)
g --> T(0.25) YZ22
Y --> D       F[+++:F]
Z --> D       F[---:F]
2 --> D       L[+++:F][---:F]
(developing of stochastic capillars)
T --> D       hT
h --> T(0.5)  ijlk
i --> D       -F[++F:h]
j --> D       +F[h]
l --> D       -F[h]
k --> D       +F[--F:h]
L --> D       :WL;
W --> T(0.5)  nmon
m --> D       :F+F-;
n --> D       :FF;
o --> D       :F-F+;
```

Comments included in brackets within the model describe parts of the human kidney generated with the corresponding production rules. Fig. 1 shows the resulting arterial system of a normal kidney. In order to obtain model variations, triangular and uniform probability distribution functions are used.

The main advantage of the program is the possibility of experimenting over the real model, by only making changes in the mathematical model description. As an example, pictures of kidney with infarct deviation and arteriosclerotic changes of the smaller blood vessels are modeled. Two approaches for generating a human kidney picture with infarct deviations are possible:

- by excluding artery production rules in the model; or
- by changing the color of infarct affected vessels.

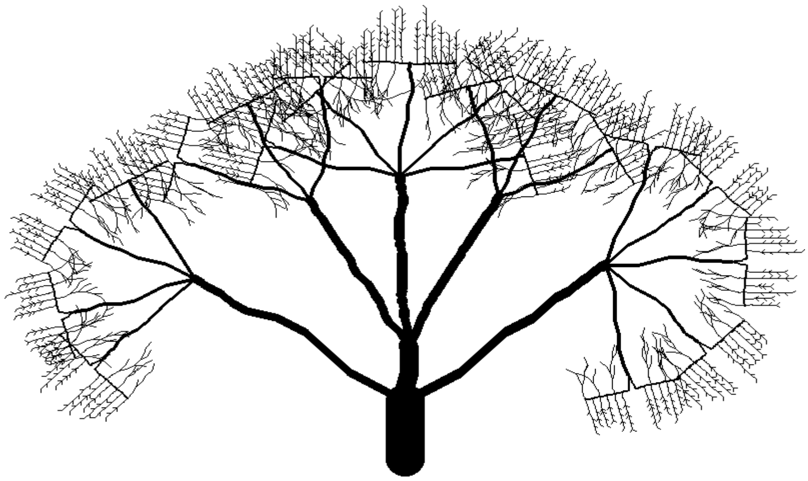

Figure 1. Picture of the arterial system on normal kidney

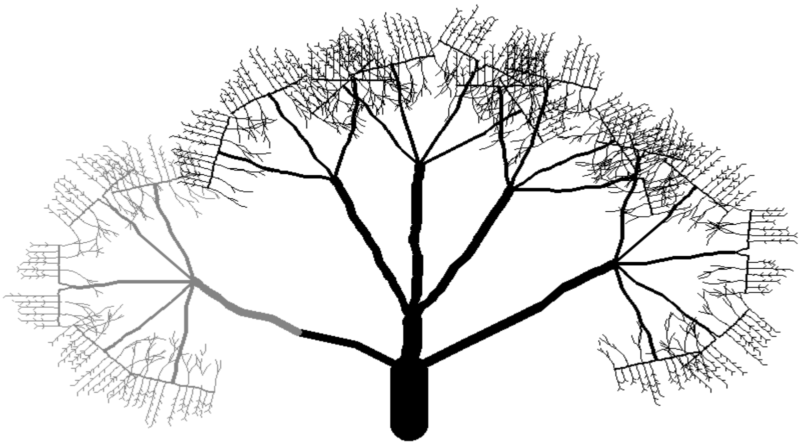

Figure 2. Picture of the arterial system on kidney with infarct deviations

In the first approach the resulting picture is a kidney with missing blood vessels which are not functional. Fig. 2 represents a kidney with infarct deviations. Different color of blood vessels in the second approach is used to present all the kidney area covered by infarct.

In a similar way the arteriosclerotic changes of the smaller blood vessels are modeled. A model which represents the vascular anatomy where 20% of the interlobular arteries are not functioning properly is generated by implementing stochastic changes in the interlobular arteries productions. The resulting model contains 20% the interlobular arteries with a different color.

5. Conclusion

An application of L-systems for generating a picture of the human kidney arterial system is presented. Stochastic L-systems are used to enable the modeling of structures with variations presented in each individual pattern. The model presented opens a new approach for presentation and investigation on anatomical changes and deviations in human body organ systems. Use of a context sensitive L-system can be considered for the future. These systems can provide information transport through the branching structure, which can be used for modeling the processes and the dynamics in the organs.

6. References

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